RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER TAKE-HOME TEST/ASSIGNMENT, MARCH 2021

THIRD YEAR [BATCH 2018-21]

Date : 16/03/2021 **MATHEMATICS (Honours)**

Time : 11am – 3pm **Paper : VI** Full Marks : 100

Instructions to the Candidates

- Write your **College Roll No, Year, Subject & Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject & Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group - A (Numerical Analysis)

Answer any 5 from question nos. 1-7 in this group. Symbols are of usual meaning. Answer to the point in your own words. [5 x 6 = 30 marks]

1. Show that the error in approximating $f(x)$ by the interpolating polynomial $L_n(x)$ is [6]

$$w(x)\frac{f^{n+1}(\xi)}{(n+1)!}\ ,\ \text{where } w(x)=(x-x_0)(x-x_1)\cdots(x-x_n).$$

2. Show that the divided difference of a function f for the distinct arguments $x_1, x_2, ..., x_n$ is given by [6]

$$f(x_1,x_2,...,x_n)=\sum_{i=1}^{n}\frac{f(x_i)}{(x_i-x_1)(x_i-x_2)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

3. Deduce Newton-Cote's formula for numerical integration. Hence, find the trapezoidal rule. [4+2]

4. Explain the fixed-point iteration method for finding a simple root of $f(x)=0$. State the sufficient condition of convergence of the method. [5+1]

5. Discuss the Gauss-Seidal iteration method for finding the solution of a system of linear equations . State the condition of convergence. [5+1]

6. Find $y(0.03)$ from the ODE : $\frac{dy}{dx}=y+x^3, y(0)=1$ taking step length $h=0.01$. [6]

7. Describe the power method to calculate the numerically greatest eigen value of a real symmetric matrix.

[6]

Group - B (Vector Calculus)

Answer any 2 from question nos. 8-10 in this group. [2 x 10 = 20 marks]

8. (a) Verify Stokes' theorem for $\overrightarrow{F}(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = (x-y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ on the upper half-sphere given by $x^2+y^2+z^2 = 2, z \geq 0$. [5]

   (b) If $\overrightarrow{f}$ and $\overrightarrow{g}$ are irrotational vector fields defined over $\mathbb{R}^3$, show that $\overrightarrow{f} \times \overrightarrow{g}$ is a solenoidal vector field. [5]

9. (a) Evaluate the following integral for $\overrightarrow{r}(t) = -5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$

$$\int_1^2 \left(\frac{1}{r}\frac{d\overrightarrow{r}}{dt} - \frac{dr}{dt}\frac{\overrightarrow{r}}{r^2}\right) dt$$

   where $r(t)$ is the magnitude of $\overrightarrow{r}(t)$. [4]

   (b) Find the values of the constants a, b and c such that the vector field

$$\overrightarrow{F}(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = [x + (1+b)y - cz]\hat{\imath} + [ax - by + (c+1)z]\hat{\jmath} + (2ax + 3by - z)\hat{k}$$

   is irrotational. [3]

   (c) Find the directional derivative of the scalar valued function

$$f(x\hat{\imath} + y\hat{\jmath}) = \begin{cases} x\sin\left(\frac{xy}{x^2+y^2}\right), & \text{if } (x,y) \neq (0,0) \\ 0, & \text{otherwise.} \end{cases}$$

   at the origin, in the direction $\beta = \hat{\imath} + \hat{\jmath}$. [3]

10. (a) Verify the divergence theorem for $\overrightarrow{F}(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = (3x-y)\hat{\imath} - 2yz\hat{\jmath} + xy^2z\hat{k}$, taken over the region bounded by $x^2+y^2 = 9, z = -1$ and $z = 3$. [6]

   (b) Find the vector equations of the tangent line and the normal plane at a point $P = (1,1,1)$ of the curve given by $xy - z^2 = 0$, $x^2 + yz - zx = 1$. [4]

Group - C (Unit I - Rigid Dynamics)

Answer question 11 and any 2 from question nos. 12 to 14 from this group. [6 + 2 x 12 = 30 marks]

11. (Compulsory) Explain the motion of a rigid body in two dimensions, defining properly the variables used which determine the motion. Find the moment of momentum of a rigid body moving in two dimensions about the centre of inertia. [2 + 4]

12. (a) Show that a uniform triangular lamina of mass M is equi-momental with three particles, each of mass $\frac{M}{12}$, placed at the angular points and a particle of mass $\frac{3M}{4}$ placed at the centre of inertia of the triangle. [7]

   (b) A solid homogeneous cone of height h and semi-vertical angle α oscillates about a horizontal axis through its vertex. Find the length of the simple equivalent pendulum. [5]

13. (a) Two equal uniform rods AB and AC are freely jointed at A and laid on a smooth horizontal table in such a way that $\angle BAC$ is a right angle. The rod AB is struck by a blow P at B in a direction perpendicular to AB. Find the initial velocity of B just after the blow. [6]

   (b) A circular homogeneous plate is projected up a rough inclined plane with velocity V without rotation, the plane of the plate being in the plane of greatest slope. Show that the plate ceases to slip after a time

$$\frac{V}{g(3\mu\cos\alpha + \sin\alpha)},$$

   where μ is the coefficient of friction and α is the inclination of the inclined plane. [6]

14. (a) Write down the general equations of motion of a rigid body in vector form. Establish the principle of independence of the motion of translation and rotation of a rigid body. [2+5]

(b) A uniform rod, of length $2a$ and weight W, is turning about its end O and starts from the position in which it was vertically above O. When it has turned through an angle θ, find the resultant reaction at O. [5]

Group - C (Unit II - Particle Dynamics)

Answer question 15 and any 2 from question nos. 16 to 18 from this group. [6 + 2 x 7 = 20 marks]

15. (Compulsory) Prove that in a parabolic orbit the time taken to move from the vertex to a point at a distance r from the focus is

$$\frac{1}{3\sqrt{\mu}}(r+l)\sqrt{2r-l}$$

where l is the semi-latus rectum. [6]

16. A spherical raindrop, falling freely, receives in each instant an increase of volume equal to λ times its surface area at that instant. Find the velocity at the end of time t, and the distance fallen through in that time. [7]

17. A particle slides from a cusp down the arc of a rough cycloid, the axis of which is vertical. Find the ratio of its velocity at the vertex will bear to the velocity at the same point when the cycloid is smooth. [7]

18. A particle moves under a force which is always directed towards a fixed point and varies inversely as the square of the distance from that point, being projected in any manner. Determine the orbit and distinguish the three cases which arise. [7]

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